Inequality and Relative Saving Rates at the Top

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Abstract

We estimate the long- and short-run relationship between top income and wealth shares for France and the US since 1913. We find strong evidence for a long-run cointegration relationship governed by relative saving rates at the top. For both countries, we estimate a decline in the relative saving rates at the top – after 1968 in France and 1983 in the US, equivalent to a reduction of the long-run gap between wealth and income inequality compared to the period before. In the short-run, income inequality drives wealth inequality, while the converse link is weaker and slower. Using counterfactual simulations, we find that the recent rise in wealth inequality in the US is largely attributable to the contemporary increase in income inequality. Modest income concentration dynamics and a stronger decline in relative saving rates at the top than in the US contributed to a more subdued rise in wealth inequality in France.

Keywords: Income inequality, wealth inequality, VECM, cointegration, top shares, saving rates.

JEL Classification: D31, E21, E25, N32, N34.

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Non technical summary

The distribution of national income and wealth are key economic variables with broad economic and societal implications. The recent development of the World Wealth and Income Database (WID), a large-scale database featuring historical cross-country inequality measures, allows for a long-term analysis of the relationship between income and wealth inequality. Income inequality and wealth inequality are related because the flow of income determines saving, which in turn determines the accumulated stock of wealth. A look at French and US data, two countries with the longest time span of data available, shows that in recent decades the share of income that goes to the top 1% or the top 10% has been rising. Simultaneously, the share of wealth owned by the top 1% and top 10% has been rising as well. Remarkably, over the last century, the top shares of income and wealth have been evolving in broadly similar ways. This paper provides an analysis of the strong co-movement of income and wealth inequality across time.

We first show theoretically that the flow of income at the top translates income inequality into stock (wealth) inequality. If individuals at the top of the distribution have a higher propensity to save than the rest of population, a given increase in income inequality (i.e. a higher share of national income going to the top of the distribution) leads to a corresponding rise in wealth inequality (i.e. a higher share of wealth going to the top of the distribution). This mechanism explains the co-movement over long time periods. In technical terms, income and wealth inequality form an error-correction relationship. We show that this relationship is governed by the relative savings rate, i.e. the savings rate of the top relative to the average savings rate in the economy.

Using the data from the WID, we estimate this relationship and recover the relative savings rate for France and the US. We find that over the last 100 years, the top 1% saves more than twice as much as the average, while the top 10% saves around 70% as much. We also empirically test the stability of these relative savings rates. We find breaks in the relationship between income and wealth inequality in both France (in 1968) and the US (in 1983). We find a decline of the relative savings rate after the break dates. This decline in the relative savings rates at the top in both France and the US imply that income and wealth inequality are closer together today than they were historically.

Using our estimated error correction relationship, we further analyze the short-run relation between wealth and income inequality. Where income inequality can causes wealth inequality through saving, the reverse causality (from wealth to income inequality) is also possible as wealth also creates (capital) income. Our results suggest that in the short-run, income inequality drives wealth inequality, while the converse effect is estimated to be rather small. Finally, we interpret the recent rise in income and wealth inequality through the lens of our estimates. We find that a large fraction of the observed rise in wealth concentration in the US in recent decades can be attributed to rising income concentration. In France, modestly increasing income inequality and a declining relative saving rate muted the rise in wealth inequality.

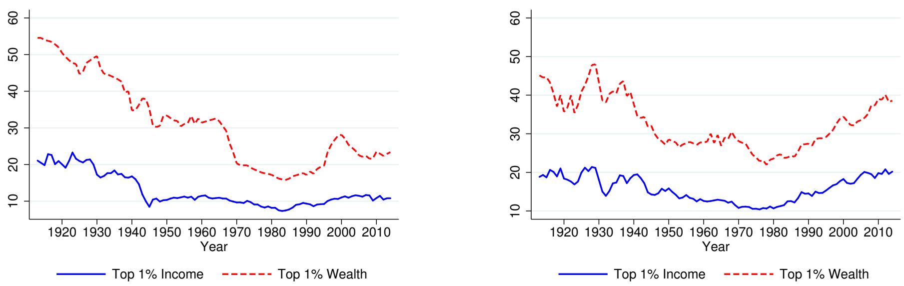
1 Introduction

Wealth and income inequality have been rising in recent decades, bringing a new impetus to inequality research. Since 1980, the income share of the Top 1% in the U.S. almost doubled to reach 20% of total national income today, and the Top 1 % wealth share increased by 13 percentage points and now amounts to 39% of all wealth. While the remarkable development of inequality itself has spurred a rich area of research, the *joint* evolution of income and wealth inequality has received limited attention.

Figure 1 depicts the historical development (1913-2014) of income and wealth inequality in France and the US, as measured by the Top 1% shares. What is most striking is the almost uniform evolution of wealth and income inequality over this long time horizon. The strong co-movement of income and wealth inequality across time – which can be observed in virtually every country – raises a number of intriguing questions. Is there a stable long-term relationship between wealth and income inequality? If so, what is the driving force of this stability over time? And how fast does income inequality lead to wealth inequality and vice versa?

Figure 1: Historical Evolution of Inequality

(a) France Top 1% **(b) US Top 1%**



Note: Top 1% shares for pre-tax national income and net personal wealth. Data from the World Wealth and Income Database.

In this paper, we aim to answer these questions and investigate the co-movement of wealth and income inequality. Our first contribution is to show that simple budget equation accounting implies an error correction relationship between wealth and income inequality. The core of this relationship is a cointegration between the top wealth share and the top income share that is stable over time as long as relative saving rates out of pre-tax income at the top are stable. Intuitively, relative saving rates at the top translate "flow" (pre-tax income) inequality into "stock" (wealth) inequality. If individuals at the top of the distribution have a higher propensity to save than the rest of population (Dynan et al., 2004), a given increase in income inequality will thus lead to a corresponding rise in wealth inequality in the long-run (Saez and Zucman, 2016). The theoretical error correction framework thus provides the basis to analyze the joint long- and short-run relationship between wealth and income inequality empirically.

The error correction framework therefore allows us to actually *estimate* the long-run relative saving rate at the top, which is our second contribution. We use annual data on top income and wealth shares for France and the US since 1913 from the Word Wealth and Income Database (WID) by Atkinson et al. (2011) and Alvaredo et al. (2013). Using (vector) error correction analysis over the whole sample 1913-2014, we estimate that the long-run relative saving rate of the Top 1% is more than twice as high as the aggregate saving rate. The estimated long-run relative saving rate for the Top 10% is lower and remarkably similar across both countries, at around 1.75. Both results are in line with the theoretical framework and with previous literature suggesting that the propensity to save increases in wealth/income (De Nardi and Fella, 2017; De Nardi et al., 2010; Dynan et al., 2004; Quadrini, 1999).

Our third contribution is to empirically test the *stability* of relative saving rates at the top over time. We find strong evidence of structural breaks in the cointegration between income and wealth concentration, in 1968 in France and 1983 in the US. In line with the structural break tests, we estimate a decline in the long-run relative saving rates (out of pre-tax income) at the top in both countries around the break dates, equivalent to a structural break in the long-run relationship between income and wealth inequality. We attribute the decline to a change in household behavior at the top, since tax progressivity did not increase in the past decades. The drop in the relative savings rates at the top in both France and the US implies that income and wealth inequality are closer together today than they were historically. Yet, from a long-run perspective, we find that the remarkably uniform co-movement between income and wealth inequality can only be explained by relative saving rates at the top that are *relatively stable* over long time periods.

As a final contribution, we use our vector error correction framework to analyze the short-run relation between wealth and income inequality. Our results suggest that in the short-run, income inequality drives wealth inequality, while the converse effect is estimated to be rather small. The absence of a strong feedback loop from wealth inequality to income inequality provides a new light on the work by Piketty (2014), in which high wealth inequality leads to higher income inequality through the returns on capital, again leading to higher wealth inequality through higher savings. Our results imply that only the effect of income on wealth is strong in the short-run, but not vice versa. Using counterfactual simulations, we find that income inequality responds rather sluggishly to higher wealth inequality. In contrast, a large fraction of the observed rise in wealth concentration in the US in recent decades can be attributed to rising income concentration. In France, modestly increasing income inequality and a declining relative saving rate muted the rise in wealth inequality. Overall, income inequality emerges as the key driver of wealth inequality in both short- and long-run, whereas lower relative saving rates at the top may only partially attenuate rising wealth inequality over the medium- and long-run.

Our paper is mainly related to the work by Saez and Zucman (2016) and Garbinti et al. (2017). Our theoretical framework is an extension of their budget equation accounting. We similarly start from the individual wealth accumulation as specified in the budget equation (where wealth is accumulated savings out of income plus capital gains) and follow their line of reasoning to show that the long-run relationship between top wealth and income shares is pinned down by the relative saving rate at the top. Saez and Zucman (2016) and Garbinti et al. (2017) discuss this relationship in a strict accounting sense, and neither discuss nor analyze the resulting implications for empirical applications. We extend their analysis by accounting for mobility across the wealth distribution and showing that the framework implies an error correction as long as the relative saving rate at the top is stable over time.

We also differ from these papers because we *estimate* the *actual long-run* relative saving rates at the top using error correction methods. In contrast, Saez and Zucman (2016) and Garbinti et al. (2017) construct annual synthetic relative savings rates as a residual from the aggregated budget equation of households. Their constructed savings rates are hence necessarily quite volatile across-time and prone to measurement error, in particular with respect to capital gains. In contrast, we estimate the long-run steady state relative savings rate as implied by the long run dynamics of the top wealth and income share, but do not derive annual savings rates. We therefore see our findings as complementary to those of Saez and Zucman (2016) and Garbinti et al. (2017).[1] As we account for wealth mobility, our error correction framework also allows us to estimate actual relative saving rates (instead of synthetic saving rates) and test their stability over time. As such, we are able to disentangle short- and long-run dynamics between income and wealth concentration and to isolate the effect of rising income inequality from a changing long-run relative saving rate to explain the recent trend rise in wealth inequality by counterfactual analyses.

Interestingly, the drop in the relative savings rate at the top we find through our error correction analysis of income and wealth inequality seems to have happened rather close in time to the drop in the aggregate savings rate. An earlier literature has investigated the puzzling large drop in the aggregate saving rates of the U.S. that started in the eighties (See among others Bosworth et al. (1991), Gokhale et al. (1996), Attanasio (1998) and Parker (1999), Browning and Lusardi (1996) provide an early overview.) While this literature could not come to a conclusion of what caused the large drop, our results are in line with more recent findings of Juster et al. (2005) that those households with higher capital gains in corporate equities reduced savings the most in the eighties. Juster et al. (2005) explain the large drop in the aggregate personal savings rate by the capital gains that households received during the stock market boom of the eighties and nineties.

[1]A possible analogy, although imperfect, is that Saez and Zucman (2016) and Garbinti et al. (2017) look at relative saving rates from a business cycle viewpoint, while we take a long-run growth perspective.

This is supported by Kuhn et al. (2018), who document the importance of portfolio compositions and asset prices on the evolution of wealth concentration using household-level data from 1949-2016 in the U.S. As large corporate equities holdings are mostly held by households at the top of the wealth distribution, our findings of a reduced relative savings rates for the top fit nicely with their findings and therefore provides indirect new evidence on why the aggregate savings rate dropped so much from the early eighties onward.

Our paper is also related to the large literature on quantitative macroeconomic models that analyze the determinants of individual and aggregate saving rates. Benhabib and Bisin (2016), Quadrini and Ríos-Rull (2015) and De Nardi and Fella (2017) survey the mechanisms and assumptions under which canonical models such as the Bewley-Huggett-Aiyagari framework are able to generate saving rate distributions that are in line with empirical observations and imply realistic wealth concentrations, in particular fat tails at the top.[2] Piketty and Zucman (2015) show that the long-run wealth concentration increases in r-g (the after-tax rate of return on wealth minus the growth rate). Slacalek et al. (2012) show that the historical evolution of aggregate saving rates in the US can be explained by a buffer-stock model of optimal consumption and saving. Hubmer et al. (2016) employ the Bewley-Huggett-Aiyagari model to analyze the rise in wealth inequality in the U.S. in recent decades and identify a substantial drop in tax progressivity as the main driver.

The rest of our paper is organized as follows. In Section 2, we discuss the theoretical relationship between wealth and income shares, which forms the basis for our econometric setup. Section 3 discusses the data and shortly describes the historical evolution of inequality. Section 4 outlines the econometric specification and presents the empirical results. In Section 5, we perform counterfactual simulations to disentangle the relative contribution of income inequality and saving rates to wealth inequality dynamics. Section 6 concludes.

2 Theoretical Framework

In order to provide a theoretical framework for our analysis, this section derives the theoretical relationship between the wealth share and the income share across the population distribution of wealth. We build on the reasoning in Saez and Zucman (2016). We extend their framework and derive an error correction relationship between income and wealth

[2] De Nardi and Fella (2017) for example survey transmission of bequests and human capital, heterogeneity in preferences and the individual rate of return, entrepreneurship, richer earning processes and medical expenses.

shares of a certain fractile f of the wealth distribution.[3]

Let W_t^i denote wealth (consisting of both non-financial and financial assets) and Y_t^i denote pre-tax income (both labor income and flow income out of wealth such as rent, interest and dividends) at time t of individual i. Then, the wealth accumulation of individual i is given by:

$$W_{t+1}^i = (1 + q_t^i)\, W_t^i + s_t^i\, Y_t^i \tag{1}$$

where q_t^i denotes the asset price growth rate (or capital gain), and s_t^i is the saving rate of individual i out of pre-tax income.[4] The first term thus captures valuation effects of existing wealth, and the second term represents the savings flow contributing to wealth accumulation. Note that the saving rate s_t^i is defined as a rate applicable to *pre-tax* income. Similarly, we can define an after-tax saving rate out of disposable income. Disposable income $Y_{d,t}^i$ is all income left after income taxation, such that we can write

$$Y_{d,t}^i \equiv (1 - \tau_t^i)\, Y_t^i \tag{2}$$

whereby τ_t^i is the average income tax rate across all income sources levied from individual i. The wealth accumulation equation can then alternatively be written as

$$W_{t+1}^i = (1 + q_t^i)\, W_t^i + s_{d,t}^i\, (1 - \tau_t^i)\, Y_t^i \tag{3}$$

where the saving rate out of disposable income is given by $s_{d,t}^i$. The relationship between the two saving rates is then given by:

$$s_t^i \equiv s_{d,t}^i (1 - \tau_t^i) \tag{4}$$

In words, the saving rate out-of pre-tax income is determined by the saving rate out of disposable income and income taxes themselves. In the following, we focus on the representation of wealth accumulation as given by Equation (1), because it allows us to derive a relationship between wealth inequality and *pre-tax* income inequality.

We measure inequality as the share of wealth and income accruing to a certain wealth fractile of the population, say the top 1 percent or the top 10 percent when ranking individuals according to their wealth level. Let us first derive the wealth accumulation of fractile f of the wealth distribution. We first define N_t^f to be the set of individuals in fractile f at time t, and s_t^f and q_t^f as their average saving rate and asset price growth

[3] When we bring this theoretical framework to the data we will use wealth share data for fractile f of the wealth distribution and income share data for fractile f of the income distribution (i.e. in the data, top income shares are not constructed from the wealthiest individuals but from the highest earners). We explain what this implies in more detail in the data section.

[4] We assume current period savings do not have a capital gain q_t^i. Saez and Zucman (2016) assume savings are made before, i.e $W_{t+1}^i = (1 + q_t^i)\,(W_t^i + s_t^i\, Y_t^i)$ and so on all current period savings there is also a capital gain. Our assumption simplifies the analysis.

rate, respectively:

$$s_t^f \equiv \sum_i^{N_t^f} s_t^i \frac{Y_t^i}{\sum_i^{N_t^f} Y_t^i} \qquad q_t^f \equiv \sum_i^{N_t^f} (1+q_t^i) \frac{W_t^i}{\sum_i^{N_i^f} W_t^i} - 1 \tag{5}$$

We may then sum the wealth accumulation Equation (1) over the individuals of fractile f to obtain the relationship between total wealth in fractile f in period t and t+1

$$\sum_i^{N_{t+1}^f} W_{t+1}^i = (1+q_t^f) \sum_i^{N_t^f} W_t^i + s_t^f \sum_i^{N_t^f} Y_t^i + c_{t+1}^f \tag{6}$$

where we control for mobility of individuals across the wealth distribution via the churn term c_{t+1}^f, defined as the difference of wealth of individuals moving into fractile f and out of fractile f in t+1:

$$c_{t+1}^f = \sum_i^{N_{t+1}^f \backslash N_t^f} W_{t+1}^i - \sum_i^{N_t^f \backslash N_{t+1}^f} W_{t+1}^i \tag{7}$$

The first term in Equation (7) represents total wealth of new entrants in fractile f at time t+1. The second term in Equation (7) represents total wealth of individuals exiting fractile f at time t+1.[5] To the extent that wealth mobility at the top of the distribution is fairly limited (Charles and Hurst (2003); Boserup et al. (2014)), there likely is substantial overlap in N_t^f and N_{t+1}^f such that the churn term is relatively small. The wealth accumulation of fractile f of the wealth distribution is then given by:

$$W_{t+1}^f = (1+q_t^f)\, W_t^f + s_t^f\, Y_t^f + c_{t+1}^f \tag{8}$$

where W_t^f and Y_t^f are wealth and pre-tax income accruing to wealth group f. The average capital gain of fractile f is given by q_t^f and s_t^f is the saving rate of fractile f. In contrast to Saez and Zucman (2016), s_t^f is the *actual* saving rate of people in wealth fractile f (in period t) and not a *synthetic* saving rate. This is because keeping track of the churn allows to explicitly account for mobility across wealth groups.[6] We will see shortly that actual saving rates are key to understand the joint evolution of wealth and income inequality.

Equivalent reasoning allows us to write the aggregate wealth dynamics for the entire

[5] In Equation (7), N_t^f and N_{t+1}^f are the sets of individuals in fractile f at time t and t+1 respectively. We have that $N_{t+1}^f = (N_{t+1}^f \backslash N_t^f) \cup (N_{t+1}^f \cap N_t^f)$ and $N_t^f = (N_t^f \backslash N_{t+1}^f) \cup (N_{t+1}^f \cap N_t^f)$, as individuals move in and out of fractile f. Individuals moving into fractile f at time t+1 are in $N_{t+1}^f \backslash N_t^f$, those moving out are in $N_t^f \backslash N_{t+1}^f$.

[6] A more detailed study of wealth distribution dynamics is beyond the scope of our paper. See Hurst et al. (1998), Charles and Hurst (2003) and Boserup et al. (2014) for studies documenting low wealth mobility in the US and Denmark, respectively. Related, Chetty et al. (2014) document a low degree of income mobility in the US and Ferrie et al. (2016) find similar evidence for educational mobility in the US. Black and Devereux (2011) provide an overview about the literature on inter-generational mobility.

population as:

$$W_{t+1} = (1 + q_t)\, W_t + s_t\, Y_t \tag{9}$$

where W_t denotes aggregate wealth and Y_t represents aggregate pre-tax income. The aggregate saving rate is given by s_t. Note that the growth rate of aggregate wealth g_t is simply the sum of the aggregate asset price growth rate and the aggregate saving ($S_t \equiv s_t Y_t$) to wealth ratio:

$$g_t \equiv \frac{W_{t+1}}{W_t} - 1 = q_t + \frac{S_t}{W_t} \tag{10}$$

To arrive at our measure of inequality, let us denote the wealth share of wealth fractile f at time t as $sh^f_{W,t}$ and its income share as $sh^f_{Y,t}$:

$$sh^f_{W,t} \equiv \frac{W^f_t}{W_t} \qquad sh^f_{Y,t} \equiv \frac{Y^f_t}{Y_t} \tag{11}$$

We can then combine Equations (8) and (9) to obtain:

$$sh^f_{W,t+1} = \frac{(1 + q^f_t) sh^f_{W,t} + s^f_t\, sh^f_{Y,t} \frac{Y_t}{W_t} + \frac{c^f_{t+1}}{W_t}}{1 + q_t + s_t \frac{Y_t}{W_t}} \tag{12}$$

This is a characterization of the wealth share of fractile f at time $t+1$ as a function of eight variables: The wealth and income share of fractile f at time t ($sh^f_{W,t}$ and $sh^f_{Y,t}$), fractile f and aggregate asset price growth rates (q^f_t, q_t) saving rates (s^f_t, s_t), the aggregate income to wealth ratio ($\frac{Y_t}{W_t}$) and churn as a fraction of aggregate wealth ($\frac{c^f_{t+1}}{W_t}$). This equation shows quite intuitively that the fractile f wealth share is *ceteris paribus* increasing in the rate of asset price changes, saving rate, churn, and income share of fractile f and is decreasing *ceteris paribus* in the aggregate asset price change and aggregate saving rate.

Finally, defining the change in the wealth share of fractile f as $\Delta sh^f_{W,t+1} \equiv sh^f_{W,t+1} - sh^f_{W,t}$ and the excess asset price growth rate for fractile f above the aggregate as $\Delta^e q_t \equiv q^f_t - q_t$, we can rewrite Equation (12) in dynamic form as:

$$\Delta sh^f_{W,t+1} = \frac{Y_t}{W_{t+1}}(s^f_t\, sh^f_{Y,t} - s_t\, sh^f_{W,t}) + \frac{\Delta^e q_t}{(1 + g_t)} sh^f_{W,t} + \frac{c^f_{t+1}}{W_{t+1}} \tag{13}$$

This equation shows the equilibrating force of deviations of the income and wealth share of fractile f. If the income share of fractile f multiplied by its saving rate rises above the wealth share of fractile f multiplied by the aggregate saving rate, the wealth share will increase (and vice versa). The wealth share also increases if there is positive excess asset price growth or positive churn, as seen from the second and third term. Following mere wealth accumulation accounting, we thus obtain a theoretical prediction on the intertemporal relationship between wealth inequality and income inequality, as represented by the shares of wealth and income accruing to a certain fractile of the population.

Let us now consider a steady state in which wealth shares, income shares and saving rates are stable. Let us furthermore assume that excess asset price growth and churn are short-run phenomena.[7] Equation (13) then implies a long-run relation between the wealth share and the income share:

$$sh_W^f = s_r^f \, sh_Y^f \tag{14}$$

with

$$s_r^f \equiv \frac{s^f}{s} \tag{15}$$

In other words, the wealth share to income share ratio for fractile f is equal to the relative saving rate s_r^f. Under the assumption of no long-run asset price growth differentials, wealth accumulation accounting thus implies a direct stable long-run relationship between wealth and income inequality, as long as relative saving rates are stable.[8] Equation (14) thus highlights the key role of saving rates in determining the long-run evolution of wealth inequality. In the long-run, the relationship between income and wealth inequality is solely governed by relative saving rates at the top. To fix intuition, suppose that all individuals had the same saving rate. Equation (14) then implies that wealth shares are equal to income shares, because everybody saves at the same rate out of income. However, if wealthy individuals have higher saving rates, relative saving rate at the top will exceed unity and the corresponding wealth shares are higher than income shares in the long-run. In combination with existing findings that saving propensity increases in wealth/income (De Nardi and Fella, 2017; De Nardi et al., 2010; Dynan et al., 2004; Quadrini, 1999), Equation (14) also implies that ratio of the share of wealth to the share of income should be larger at higher levels of the distribution (i.e. larger for Top 1% than for the Top 10%).

The relative savings rate and income inequality are therefore two important long-run determinants of wealth inequality. For instance, keeping the relative saving rate constant, an increase in income inequality leads to higher wealth inequality. Similarly, an increase in the relative saving rate, holding constant income inequality, leads to higher wealth

[7] In a steady-state without wealth mobility, the churn term is zero. A constant positive steady-state level of churn would only alter the long-run relationship up to a constant.

[8] When allowing for non-zero long-run asset price growth differentials, the long-run relationship between wealth and income inequality is given by

$$sh_W^f = s_r^f \left(1 - \frac{W}{S}\Delta^e q^f\right)^{-1} sh_Y^f \tag{16}$$

such that the long-term link is not solely captured by saving rates only. The term in brackets is equal to one under no asset price differentials. From a theoretical perspective, if this assumption is not fulfilled, there can only be one asset in the economy. It is thus intuitive to assume that all assets grow at the same rate in the long-run, with the rate being equal to consumer price inflation such that the wealth-to-income ratio does not go to infinity (Garbinti et al., 2017).

inequality. This relationship is stable over time as long as the steady state relative saving rate is unchanged.

We now show that using the assumption of the existence of a steady state relative savings rate and savings to wealth ratio, we can derive an error correction relationship between the wealth share, income share and the steady state relative savings rate. We first define u_t and v_t as the time t deviations from steady state of the savings to wealth ratio and relative savings rate.

$$-\frac{S_t}{W_{t+1}} = -\frac{S}{W} + u_t \tag{17}$$

$$-s^f_{r,t} = -s^f_r + v_t \tag{18}$$

We can now rewrite Equation (13) as the sum of a steady state cointegration relationship and temporary deviations,

$$\Delta sh^f_{W,t+1} = -\frac{S}{W}(sh^f_{W,t} - s^f_r sh^f_{Y,t}) + \frac{c^f_{t+1}}{W_{t+1}} + e_t \tag{19}$$

where the error term is given by

$$e_t = \left[sh^f_{W,t} - s^f_{r,t} sh^f_{Y,t}\right] u_t + \left[sh^f_{Y,t}(-\frac{S}{W})\right] v_t + \left[\frac{sh^f_{W,t}}{(1+g_t)}\right] \Delta^e q^f_t \tag{20}$$

Note that if the temporary deviations from the steady state and the excess capital gain are zero in expectation, it holds that

$$E\left[e_t\right] = E\left[u_t\right] = E\left[v_t\right] = E\left[(\Delta^e q^f_t)\right] = 0 \tag{21}$$

such that Equation (19) is an error correction. The cointegration relationship is governed by the steady state relative saving rate, s^f_r. The short-run adjustments are captured by the term in front of the cointegration equation, the steady state savings to wealth ratio $-\frac{S}{W}$. Intuitively, the larger steady state aggregate saving relative to wealth, the faster the adjustment, i.e. the stock of wealth adjusts faster when savings are a larger fraction of wealth. Wealth mobility as captured by $\frac{c^f_{t+1}}{W_{t+1}}$ causes temporary fluctuations of the wealth share. The last term e_t represents temporary deviations from steady state caused by aggregate savings to wealth fluctuations u_t, relative saving rate fluctuations v_t and asset price effects $\Delta^e q^f_t$ which are present if the magnitude of capital gains from fractile f differ from the aggregate. In our empirical analysis, Equation (19) is the key relationship we want to test.

The key insight here is that we can use data on top wealth and income shares to estimate the steady state relative saving rates at the top as the key driver of wealth inequality in the long-run from the data. To illustrate, assume the US is in steady state, then the current wealth and income share of the top 1% (at 38.6% and 20% respectively) would imply that the wealthy save at a rate almost twice as much as the general population (i.e. 38.6/20=1.93). Estimating Equation (19) allows us to give answers to a number of questions: What relative saving rate does the data imply? Is there a stable long term relationship between wealth and income inequality which would be implied by a stable steady state relative saving rate? What are the short run dynamics between those two, and in particular how fast does income inequality lead to wealth inequality and vice versa?

The next section presents the data used for the empirical analysis and provides a short description of the historical evolution of wealth and income inequality.

3 Data

We interpret the framework outlined above as providing a theoretical dynamic link between income and wealth inequality. In other words, in line with much of the literature, we take income and wealth shares accruing to a certain fraction of the population as inequality measures. While there are many different ways to measure inequality, this measure follows directly from straightforward wealth accumulation accounting and allows us to capture both long- and short-run relationship between income and wealth.

Up to recently, the joint dynamics of wealth and income inequality could not be studied due to a lack of long time series with annual observations. We use data from the World Wealth and Income Database (WID), a large-scale database featuring historical cross-country inequality measures resulting from the seminal contributions by Atkinson, Piketty and Saez.[9] Notably, it contains historical income and wealth shares that are available exhaustively over larger time spans, with the concepts and definitions being harmonized across time and countries.[10]

In our analysis, we focus on the relationship between income and wealth inequality in France and the US. This permits a straightforward comparative study among two countries which have been the subject of many previous studies, in particular seen individually. While both France and the US are industrialized countries, they are however heterogeneous in terms of taxation history, financial markets, demographics and many other factors that may affect income and wealth inequality. As documented by e.g. Alvaredo et al. (2013) and Saez and Zucman (2016) and described in more detail further

[9] See Piketty (2003), Piketty and Saez (2003), Atkinson et al. (2011), Alvaredo et al. (2013), Piketty and Zucman (2014a),Piketty and Zucman (2014b) and Atkinson and Piketty (2014a,b).

[10] Alternative inequality indicators such as the Gini coefficient suffer from limited data availability and cross-country comparability, rendering them unattractive for our analysis. Gini coefficients are for example collected by UNU-WIDER, see `https://www.wider.unu.edu/`. The earliest available observations for France and US are 1956 and 1944, respectively.

below, the historical evolution of income and wealth inequality differs markedly across these countries. To the extent that our empirical analysis identifies similar long-run relationships between income and wealth inequality across countries, we may interpret this as evidence for a stable relationship largely independent of short-term policies.

As with the choice of the inequality indicator itself, one further reason for choosing these particular countries is the excellent availability of data. For the US, we use yearly data from 1913-2014 from the WID. Earlier data for France is partially available, but scattered across years. We want to avoid having to heavily rely on interpolation or multiple imputation to fill data gaps, as doing so may generate noise in the estimation of the short-run relationships. We hence limit our effective sample size for France to 1913-2014 as well, such that the time span considered is harmonized across countries.

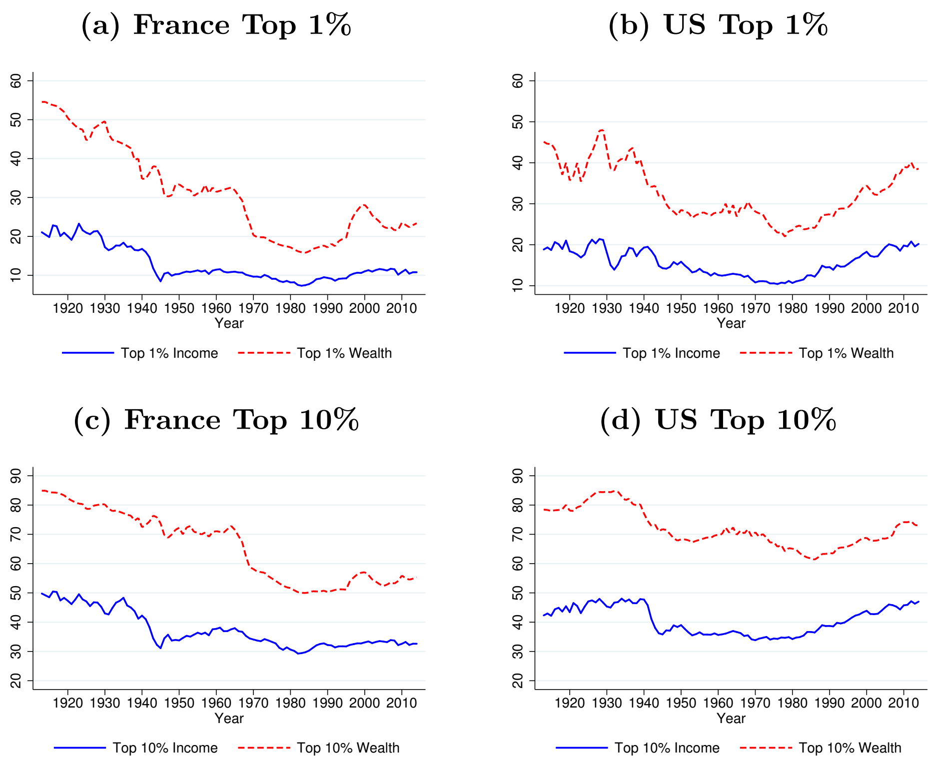
As baseline measures of income inequality, we hence use the shares of pre-tax national income accruing to the Top 1% and Top 10% of the income distribution. For international comparisons, using national income is advisable since alternative definitions such as fiscal income vary across countries. National income comprises both labor income and capital income as well as pensions. We prefer to use the total income instead of just labor income because it allows us to capture the entire flow aspect of inequality. Not taking capital income into account would be equivalent to ignoring about 30% of total national income and disregarding a source of income that is particularly relevant at the top of the distribution. Measuring income inequality before taxes and transfers allows us to trace out the redistributive effects of the tax system across countries.

For wealth inequality, we use the shares of net personal wealth held by the Top 1% and Top 10% of the wealth distribution. Net personal wealth is defined as the sum of non-financial (housing, land...) and financial (deposits, bonds, equities) assets owned by an individual, minus its personal debt. The data for France data is from Garbinti et al. (2017), US data is from Piketty et al. (2016, 2018b).[11] As for income inequality, our main focus is on the Top 1% and Top 10% since these measures are straightforward to interpret. At the same time, focusing on the Top 1% and Top 10% strikes a balance between capturing inequality evolution at the tail and being less subject to measurement errors that are likely more severe for higher percentile shares (say Top 0.1% or Top 0.01%).

Figure 2 shows the historical evolution of income and wealth inequality in France and the US, for both the Top 1% and the Top 10% shares. The upper panels mirror Figure 1. There is a clear downward trend for both income and wealth inequality in both countries, starting around 1920 in France and post-WW2 in the US. The decline in inequality lasts until the 1970s, approximately, with both countries experiencing a substantial final sharp decline of wealth concentration around the 1970s. Since then, the US experienced a steady upward trend of inequality, almost reaching historical peak levels

[11] For further details on the exact definitions, concepts and calculation methods underlying the income and wealth inequality measures employed here see Alvaredo et al. (2016).

Figure 2: Historical Evolution of Inequality



Note: Top 1% and Top 10% shares for pre-tax national income and net personal wealth. Data from the World Wealth and Income Database.

in 2014. Inequality in France similarly increased after the 1970s until today, although not as pronounced as in the US. For both countries, there is a substantial temporary increase of wealth concentration during the dot-com-bubble in the late 1990s. What is most striking is the remarkably close long-run co-movement of wealth and income inequality when measured by top shares.

It should be noted that our theoretical framework implies a link between wealth and income shares at the top of the *wealth* distribution. In our empirical analysis, however, we use the income shares at the top of the *income* distribution. It is important to recognize that, unfortunately, joint distributions of income and wealth over very long periods are difficult to obtain. The only study we are aware of is a very recent contribution by Kuhn et al. (2018), who assemble household-level data from 1949-2016 for the U.S. Such data allows to construct income share of certain wealth groups, e.g. the income share of the top 1 percent wealth holders. Over even longer time periods, the closest existing data is top wealth share data constructed from wealth distribution data and top income shares constructed from income distribution data, as available from the WID. In practice, the top 1 percent wealth holders do not necessarily coincide with the top 1 percent income earners – although the overlap might be substantial and is probably even more considerable for the top 10 percent. Kuhn et al. (2018) show that the income share of the Top 1% and the Top 10% of the wealth distribution are a bit lower than the income share of the Top 1%

and the Top 10% of the income distribution. Importantly for our analysis, however, they document that the overall pattern and evolution over time are highly similar. This allows us to extract long-run trends in the joint evolution of wealth and income inequality despite the data limitations. Nevertheless, this caveat should be kept in mind when interpreting the absolute magnitude of our estimated relative saving rates.

4 Empirical Results

In this section, we present and discuss our main empirical results on the joint short- and long-run evolution of income and wealth inequality. We start by presenting and discussing the estimation results using the whole sample period. We then perform structural break tests and sample splits to investigate the stability of the estimated relative saving rates at the top. Lastly, we decompose the determinants of the relative saving rates.

4.1 The Long- and Short-Run Relationship Between Wealth and Income Inequality

As seen in the previous section, the historical evolution of income and wealth inequality shows large changes over time. However, the remarkably uniform development of income and wealth inequality seen in the data is in line with the theoretical framework and suggests that there may exist a stable long-run relationship. We therefore explore whether income and wealth inequality are cointegrated. As a basis for the cointegration analysis, we start by performing (augmented) Dickey-Fuller tests to check whether our inequality indicators are non-stationary. The augmented Dickey-Fuller test fails to reject the null hypothesis of a unit root for both income and wealth shares, regardless of the selected lag length and in both countries. The results are shown in the Appendix in Table A1. We therefore conclude that our preferred measures of income and wealth inequality feature a unit root, rendering cointegration analysis appropriate.

Econometric Setup: Our main equation of interest is the dynamic relationship between wealth shares and income shares at the top, as derived in Section 2:

$$\Delta sh^f_{W,t+1} = -\frac{S}{W}(sh^f_{W,t} - s^f_r sh^f_{Y,t}) + \frac{c^f_{t+1}}{W_{t+1}} + e_t \tag{22}$$

which states that the change of fractile f's wealth share $sh^f_{W,t+1}$ is a function of the current wealth share, the current income share $sh^f_{Y,t}$, the relative saving rate s^f_r, short-term fluctuations caused by churn and some temporary deviations e_t.

We are primarily interested in estimating the relative saving rate s^f_r governing the long-run relationship between income and wealth inequality. For each country (France, US) and both measures of inequality (Top 1% share, Top 10% share) we use three different approaches to estimate relative saving rates at the top: (a) a Vector Error Correction Model (VECM), (b) Canonical Correlation Regressions (CCR) following Park (1992) and (c) standard Ordinary Least Squares (OLS) regressions.

For the VECM model, the empirical specifications is given by

$$\begin{pmatrix} \Delta sh^f_{W,t} \\ \Delta sh^f_{Y,t} \end{pmatrix} = \begin{pmatrix} \alpha_W \\ \alpha_Y \end{pmatrix} \begin{pmatrix} 1 & -s^f_r \end{pmatrix} \begin{pmatrix} sh^f_{W,t-1} \\ sh^f_{Y,t-1} \end{pmatrix} + \sum_{i=1}^{p-1} \begin{pmatrix} \gamma_{WW,i} & \gamma_{WY,i} \\ \gamma_{YW,i} & \gamma_{YY,i} \end{pmatrix} \begin{pmatrix} \Delta sh^f_{W,t-i} \\ \Delta sh^f_{Y,t-i} \end{pmatrix} + \begin{pmatrix} \varepsilon_{W,t} \\ \varepsilon_{Y,t} \end{pmatrix} \tag{23}$$

where $\mathbf{sh^f_t} = (sh^f_{W,t} \; sh^f_{Y,t})'$ contains the wealth shares $sh^f_{W,t}$ and income shares $sh^f_{Y,t}$. The first VECM equation captures Equation (22) – as we do not observe churn and as e_t will clearly be autocorrelated and heteroskedastic, we capture the short-term fluctuations by lags in income and wealth share changes. The second equation allows us to account for the contemporaneous feedback of wealth concentration on income concentration. Given that our measure of income contains capital income, this is one approach to control for possible simultaneity issues. Regarding the long-run relationship, this VECM setup allows for up to one cointegration relationship only, as we consider the joint evolution of two time series at a time – for example income and wealth shares accruing to the Top 1% in France. Identification of the cointegration relationship is straightforward by using the Johansen normalization. The VECM hence yields the estimated relationship

$$sh^f_W - \widehat{s}^f_r sh^f_Y \sim I(0) \tag{24}$$

and thus an estimate of the long-run relative saving rate at the top, which can be interpreted as empirical test of the error correction representation in Equation (22). As byproduct of the VECM, we also obtain estimates of the parameters α_W and α_Y which capture the short-run adjustment from imbalances in the cointegration relationship on wealth and income inequality, respectively. The VECM hence allows us to characterize the *joint short- and long-run* evolution of income and wealth inequality by disentangling short- and long-run adjustments.

In addition to the VECM, we can also obtain estimates of the long-run relative saving rates only by using the CCR estimator and OLS. Essentially, these two approaches consist of estimating – for each country and each measure of inequality separately – a single equation given by:

$$sh^f_{W,t} = s^f_r \, sh^f_{Y,t} + \varepsilon_t \tag{25}$$

If wealth and income concentration are cointegrated, the simple OLS estimator of s^f_r is super-consistent. However, in the present context, it is likely that there are common exogenous forces driving both income and wealth shares simultaneously. This is equivalent to a non-zero correlation between shocks that only affect income concentration and

ε_t, in which case the OLS estimates features non-Gaussian asymptotically biased and asymmetric distributions, invalidating standard inference. We therefore employ the CCR approach proposed by Park (1992) that uses a semiparametric correction to overcome these problems.[12] The CCR estimator is asymptotically unbiased and has fully efficient normal asymptotics, allowing for standard Wald tests using asymptotic χ-squared statistical inference.[13] The main drawback of CCR and OLS is that they are by construction silent about short-run adjustments between income and wealth inequality.

We assume that there are no linear time trends in the income and wealth shares per se. On the one hand, this is motivated by graphical inspection, which suggests the absence of linear time trends over the entire sample. On the other hand, this is also in line with the theoretical framework outlined above. We therefore also abstract from time trends and a non-zero constant mean in the cointegration equation. For the VECM setup, the appropriate lag length p is chosen using the information criteria by Schwarz (SBIC), Hannan and Quinn (HQIC) and Akaike (AIC), as reported in Table A2 in the Appendix.[14]

Baseline Results: Based on this econometric specification, our baseline results for France and the US, using as inequality measures the share of wealth and income accruing to the Top 1% or Top 10% over the entire sample period 1913-2014, are presented in Table 1:

The estimates for the relative saving rates s_r^f as the cointegration relationship parameters are highly significant and remarkably similar across estimators, indicating stationary linear combinations of income and wealth inequality. For example using the Top 1% shares in France, we obtain from the VECM that:

$$sh_W - 2.34\, sh_Y \sim I(0) \tag{26}$$

In other words, all estimators imply a statistically significant long-run relationship between income inequality and wealth inequality in both countries and for both top shares. For the Top 1%, the estimated relative saving rate in France is at 2.34 for the VECM, considerably above the one for the US which is given by 2.03. In terms of long-run effects, this means that an increase of the Top 1% income share by 1 percentage point is associated with an increase of the Top 1% wealth share of 2.34 percentage points (pp) in

[12] Essentially, the CCR procedure transforms both regressand and regressor using estimates of one- and two sided long-run covariance-matrices between ε and regressor innovations. The estimates are then obtained by applying OLS to the transformed variables.

[13] Alternative estimators are the fully-modified OLS by Phillips and Hansen (1990) and dynamic OLS by Saikkonen (1992). In our analysis, we found only minuscule differences in both estimates and standard errors compared to the CCR.

[14] The lag length tests favor parsimonious models, with for example both SBIC and HQIC indicating $p = 1$ for the Top 1% shares in the US. For the remaining specifications, we strike a balance between the information criteria by setting $p = 2$.

Table 1: Baseline Results

	France		US	
	Top 1%	Top 10%	Top 1%	Top 10%
Relative Saving Rate s_r^f				
VECM	2.34***	1.74***	2.03***	1.73***
	(0.10)	(0.06)	(0.07)	(0.06)
CCR	2.40***	1.75***	2.07***	1.77***
	(0.09)	(0.05)	(0.03)	(0.04)
OLS	2.44***	1.76***	2.07***	1.76***
	(0.03)	(0.01)	(0.02)	(0.01)
Short-Run Adjustments				
α_W	−0.09***	−0.04**	−0.10**	−0.04**
	(0.03)	(0.02)	(0.05)	(0.02)
α_Y	0.03	0.03	0.01	0.01
	(0.02)	(0.02)	(0.03)	(0.02)

Note: Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

France (and 2.03 pp in the US). For the Top 10% shares, the estimated long-run relative saving rates and effects are lower and quite similar across countries at 1.74 and 1.73, respectively.

Our long-run estimate of the relative savings rates from the cointegration relationship for the Top 1% and 10% in the US and France can be compared with the results obtained by Saez and Zucman (2016) and Garbinti et al. (2017), who back out (relative) saving rates as residual from the budget equation.[15] Our estimate for the Top 1% (Top 10%) for the US at 2.03 (1.73) is between the median, 2.6 (2.3), and mean, 1.7 (1.4), of the constructed relative savings rates for the period 1917-2011 in Saez and Zucman (2016). Similarly, for France our estimate for the Top 1% (Top 10%) at 2.34 (1.74) is close to the median, 2.37 (1.94) of the annual relative savings rates over the period 1970-2013 in Garbinti et al. (2017).[16] We believe this finding strengthens the case for our interpretation of the estimates from VECM, CCR and OLS as measuring the *long-run* relative savings rate.

[15]Saez and Zucman (2016) provide annual constructed relative savings rates from 1917 to 2011 in their Online Appendix in TableB33b. For a given fractile f, they first compute the asset price effects q_t^f using aggregate data and then construct the absolute savings rate so that the budget equation of the fractile holds. Relative savings rates are then simply the ratio of absolute savings rate of the fractile relative to the aggregate saving rate. In a similar vain, Garbinti et al. (2017) construct annual aggregate and savings rates by wealth group for the period 1970-2013 in their Online Appendix Table B8. From their constructed savings rates we calculate annual relative savings rates.

[16]From their constructed savings rates in Online Appendix Table 8 we calculate annual relative savings rates for the period 1970-2013 and then take the mean and median. The mean of the relative savings rates for the Top 1% (Top 10%) is 3.34 (2.20). The means are substantially larger than the medians as they are affected by large positive outliers. Note also that Garbinti et al. (2017) do not provide savings rates by wealth fractile before 1970.

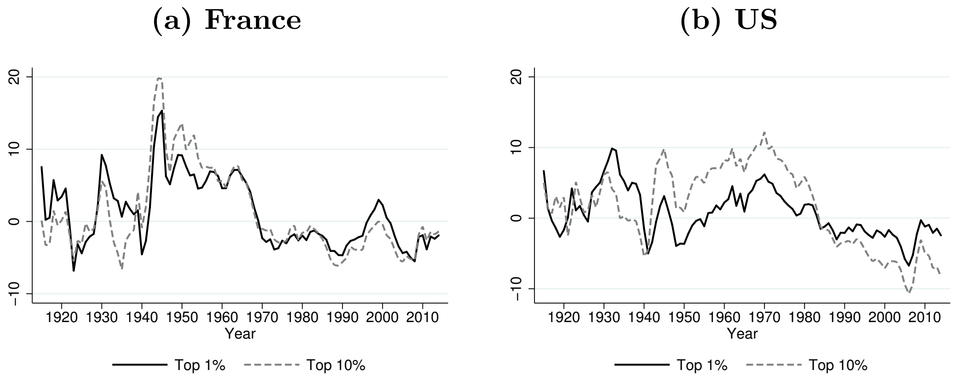
Turning to the short-run adjustments in the VECM, the parameter α_W is estimated to be statistically significant and around the same order of magnitude across the two countries. In the VECM, α_W governs the short-run change in wealth inequality as a reaction to imbalances in the long-run relationship. To fix intuition, suppose that the share of Top 1% income is high relative to wealth inequality, such that the cointegration equation is negative. The negative and statistically significant coefficient α_W then implies that the negative imbalance between the Top 1% shares creates a short-run adjustment of the wealth share. When the Top 1%'s share of income increases by one percentage point above its long-run-consistent value, the Top 1% share of wealth quickly rises as well by about approx. $0.09 \cdot 2.34 = 0.21$, about 10% of the long-run effect. This suggests that short-run changes in income inequality are associated with rather rapid short-run changes in wealth inequality. However, the effect is more pronounced for the Top 1% shares than for the broader measure of inequality.

For the converse case, the estimates of the short-run effect α_Y from wealth inequality to income inequality, have positive signs. This is in line with expectations. Let us suppose that wealth inequality is high relative to income inequality, in which case the cointegration equation is positive. A positive α_Y then implies that the high wealth inequality leads to a short-run increase in income inequality. However, the estimated coefficients are not statistically significant and considerably smaller than the estimates of α_W. This suggests that the short-run link from wealth inequality to income inequality is, if anything, weaker than the reciprocal effect. In turn, we may conclude that wealth inequality leads to higher income inequality considerably slower in the long-run. This is intuitive remembering that income inequality refers to a flow concept, while wealth inequality is about a stock. We believe this finding to be quite important in the discussion of the evolution of wealth inequality and on possible policy interventions. It suggests wealth inequality is much driven by income inequality, but is not providing a strong feedback loop back into income inequality. We investigate the short-run dynamics in more detail in Section 5.

Given these results, we can also take a closer look at the historical evolution of inequality through the lenses of our VECM. Figure 3 shows the estimated cointegration relationship and associated deviations of top shares from the long-run equilibrium:

At the beginning of our sample, the estimated cointegration relationship is mainly positive, indicating that wealth inequality was large relative to income inequality. The statistically significant negative α_W indicates that such an imbalance leads to a decrease of wealth concentration in the short-run, in line with the empirical observation: In both France and the US, substantial decreases of wealth inequality occur around 1930 (coinciding with the Stock Market Crash of 1929) and 1939-1945 (the World War II years). At the same time, the VECM captures the high volatility in inequality during that period of time. After 1945, the positive imbalance in the cointegration equation persists, in line with the observed further decline of wealth inequality during that period of time.

Figure 3: Inequality Imbalances



Note: The plots show the estimated cointegration relationship $sh_W^f - \widehat{s}_r^f sh_Y^f$ resulting from the VECM given in Equation (23).

With income inequality remaining broadly stable or decreasing further until the 1970s, the trend of falling wealth concentration continued until the 1970s, approximately.

In France, both income and wealth shares settled somewhat after the 1970s and increased modestly thereafter, as indicated by the cointegration relationship being close but below zero. This suggests that France experienced a slight upward pressure of income inequality on wealth inequality in past decades. However, this effect is considerably more pronounced in the US, where the estimated cointegration relationship quickly decreases after 1970 and stays negative until today. With the exception of a short interruption during the climax of the global financial crisis, the VECM results mirror the steadily upward trending income inequality in the US and the associated snowballing effect on wealth inequality.

Summarizing the results from this section, we find evidence of a snowballing effect of income inequality on wealth inequality, in line with the argument by Saez and Zucman (2016). Higher income inequality elevates wealth inequality considerably in the short-run. In turn, a larger concentration of wealth at the top of the distribution translates into rising capital income. The latter effect is not particularly rapid and strong in the short-run. It is however highly relevant in the long-run, as we provide evidence for a long-run link between income and wealth concentration. Because relative saving rates at the top exceed unity, a rise in income inequality yields an even higher increase in wealth inequality in the long-run.

4.2 Structural Breaks of Relative Saving Rates at the Top

Our baseline VECM analysis provides evidence that there exists a long-run relationship between income and wealth inequality in both France and the US. We now investigate the stability of relative saving rates at the top, which govern the long-run link. On the one hand, this is motivated by graphical inspection of the raw inequality time series presented in Section 3. While the "distance" between income and wealth concentration (which can be seen as a rule-of-thumb measure of the relative saving rate) is quite stable over time, it is by no means perfectly constant. On the other hand, the changed path in the historical evolution of inequality suggests that there may be structural breaks in long-run relative saving rates at the top. Post-estimation diagnosis of the VECM using the whole sample period furthermore suggests that stability may be concern: For all specifications, the Johansen (1995) Lagrange multiplier test indicates residual autocorrelation, and for the Top 10% shares in the US, one eigenvalue is relatively close to 1 (compare Section C in the Appendix).

Testing for Stability: We thus check for the existence of a *stable* cointegration relationship between income and wealth inequality by using the canonical Johansen (1991) test. For most specifications, the Johansen cointegration test indicates a VECM rank of 0, as shown Table A4 in the Appendix. In other words, the Johansen test fails to detect a *stable* long-run relationship between income and wealth inequality. In turn, this suggests that relative saving rates at the top may not be entirely stable over time due to a structural break.

To detect the structural break dates of the relative saving rates at the top, we employ standard supremum Wald tests (Andrews, 1993; Hansen, 1997) and cumulative sum stability tests (Brown et al., 1975; Ploberger and Krämer, 1992). Both testing procedures indicate the presence of a structural break in the long-run relationship between income and wealth inequality occurring in 1968 in France, and in 1983 in the US. This suggests that long-run relative saving rates at the top may have changed substantially around that point in time, both when looking at the Top 1% and the Top 10%.[17]

Table 2: Structural Breaks in Relative Saving Rates

	France		US	
	Top 1%	Top 10%	Top 1%	Top 10%
Supremum Wald	1968***	1968***	1983***	1983***
	(0.00)	(0.00)	(0.00)	(0.00)
Cumulative sum	1968***	1968***	1983***	1983***
	(0.00)	(0.00)	(0.00)	(0.00)

Note: Estimated break dates of relative saving rates at the top. p-values in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Equivalently, this implies that the estimated relative saving rates presented in the previous section may not be stable over time, as also implied by the Johansen test. We feed the estimated break dates into the Pesaran et al. (2001) procedure to test whether

[17]We also perform univariate structural break tests on the individual time series. Table A3 in the Appendix shows that univariate structural breaks occur at around the same time as the breaks in the relative saving rate.

income and wealth inequality are nevertheless cointegrated after accounting for the structural break. That is, we estimate single-equation autoregressive distributed lag (ARDL) models for income and wealth shares, whose error correction model (ECM) representation is given by

$$\Delta sh^f_{Y,t} = \alpha(sh_{Y,t-1} - \beta sh_{W,t-1}) + \sum_{i=1}^{p-1} \gamma_{Y,i} \Delta sh^f_{Y,t-i} + \sum_{i=0}^{q-1} \gamma_{W,i} \Delta sh^f_{W,t-i} + d_t + e_t \qquad (27)$$

Note that we include dummies d_t equal to 1 after and including 1968 for France and 1983 for the US to account for the structural breaks.

The hypotheses equivalent to no cointegration are

$$H_0^F : \alpha = 0 \cap \beta = 0 \qquad H_0^t : \alpha = 0 \qquad (28)$$

where α is the short-run speed-of-adjustment coefficient and β is the long-run coefficient β.[18] Following Pesaran et al. (2001), the existence of a long-run relationship is confirmed if both H_0^F and H_0^t are rejected. Table 3 shows the results, which indicate that the null hypothesis of no cointegration can be rejected at conventional confidence levels in all four cases after accounting for the structural breaks. This constitutes evidence that a long-run relationship between income inequality and wealth inequality exists, yet is not entirely stable because relative saving rates at the top were subject to structural breaks.

Table 3: Pesaran-Shin-Smith Cointegration Tests

	France		US	
	F-test	t-test	F-test	t-test
Top 1%	6.24***	-3.18**	7.66***	-3.91***
Top 10%	4.62**	-2.60**	5.58**	-3.33***
10% crit. value	3.28	-2.28	3.28	-2.28
5% crit. value	4.11	-2.60	4.11	-2.60
1% crit. value	6.02	-3.22	6.02	-3.22

Note: Critical values from Pesaran et al. (2001) using the whole samples and including time dummies according to structural break tests. For $I(1)$ regressors, H_0^F is rejected if F is larger than the critical value, and H_0^t is rejected if t is smaller than the critical value.
$p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Sample Splits: Given the results from the structural break tests and the Pesaran-Shin-Smith cointegration test, we split our sample at the estimated break dates in 1968

[18]The ARDL equation is estimated by OLS. Estimates of the short-run parameter are $\sqrt{T}$-consistent, while the estimate of the long-run parameter is super-consistent if the variables are $I(0)$. Both estimates are asymptotically normally distributed irrespective of the order of integration (Shin and Pesaran, 1999). The lag length of the ARDL equations is chosen according to the information criteria by Schwarz and Akaike.

for France and 1983 for the US. We then re-estimate our baseline regressions outlined above for the three estimators VECM, CCR and OLS. Table 4 shows the corresponding estimates of relative saving at the top.

Table 4: Estimated Relative Saving Rates $\widehat{s}_r^f$

		France			US		
		1913-2014	1913-1967	1968-2014	1913-2014	1913-1982	1983-2014
Top 1%	VECM	2.34***	2.53***	2.12***	2.03***	2.14***	1.90***
		(0.10)	(0.11)	(0.06)	(0.08)	(0.06)	(0.02)
	CCR	2.40***	2.54***	2.11***	2.07***	2.16***	1.90***
		(0.09)	(0.10)	(0.02)	(0.03)	(0.02)	(0.01)
	OLS	2.44***	2.54***	2.13***	2.07***	2.16***	1.90***
		(0.03)	(0.04)	(0.03)	(0.02)	(0.03)	(0.02)
Top 10%	VECM	1.74***	2.01***	1.65***	1.73***	1.88***	1.53***
		(0.06)	(0.10)	(0.01)	(0.06)	(0.04)	(0.03)
	CCR	1.75***	1.82***	1.65***	1.77***	1.84***	1.60***
		(0.05)	(0.06)	(0.01)	(0.04)	(0.04)	(0.01)
	OLS	1.76***	1.81***	1.66***	1.76***	1.83***	1.61***
		(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)

Note: Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

The estimated long-run relative saving rates continue to be estimated highly significantly across samples. Moreover, all three estimators present strong evidence of a decline in relative saving rates after the structural breaks. In the first sample, relative saving rates at the top are about 2.54 (1.82) for the Top 1% (Top 10%) in France and around 2.16 (1.84) in the US. This is equivalent to roughly 1-2 VECM standard errors above the whole sample results. In the second sample, relative saving rates decline compared to the first sample in both countries and both measures of inequality. The drop is particularly pronounced for the Top 1% in France, where the relative saving rates decreased by about 0.4 for the Top 1%. The relative saving rates of the Top 10% in France decreased by 0.15 after 1968. In the US, the drop is comparable in size for Top 1% and Top 10%, at roughly 0.25.

While these results are associated with the caveat of a substantially smaller sample size, they support the structural break tests presented above.[19] When performing a test for equality of relative saving rates across samples, simple Wald tests strongly reject the null hypothesis of equality in all cases. We therefore find evidence that long-run relative saving rates at the top declined considerably and statistically significant around the 1970s/1980s in both France and the US.

We may summarize our empirical findings as follows. First, we find strong econometric evidence of a long-run cointegration relationship between income and wealth inequality.

[19] The estimates of the short-run adjustment parameters are especially vulnerable to the small sample size. Section D in the Appendix provides more details of the short-run coefficient estimates in the two subsamples and some economic interpretation.

This holds for both countries and both measures of inequality. Second, the associated estimated long-run relative saving rates is higher for the Top 1% than for the Top 10%, in line with theoretical considerations. Third, sample splits yield some evidence that the long-run relative saving rates decreased after the structural breaks in both countries in the 1970s/1980s. Fourth, short-run adjustments run mainly from income to wealth inequality, while the reciprocal effect is considerably slower. Overall, these findings underline the theoretical prediction from Equation (13): To understand the historical evolution of wealth inequality, it is crucial to consider income inequality dynamics and changes in relative saving rates at the top.

Why did the relative saving rates decline? To understand why the relative savings rates declined in France and the US, we first check whether taxation could be the culprit. The estimated relative saving rate are out of pre-tax income, which can be decomposed into a relative savings rate out of disposable income and a relative tax ratio:

$$s_r^f = \frac{s_d^f}{s_d} \frac{1-\tau^f}{1-\tau} \tag{29}$$

where s_d^f is the savings rate of fractile f out of disposable income, s_d the aggregate savings rate out of disposable income, τ^f the average income tax rate of fractile f and τ the aggregate average tax rate. A drop in the relative savings rate s_r^f could then e.g. be caused by a decline in the relative saving rate out of disposable income ($\frac{s_d^f}{s_d}$). Alternatively, even with constant savings rates out of disposable income, an *increase* in the progressivity of the tax system (i.e τ^f increases relative to τ) *reduces* the relative tax ratio $\frac{1-\tau^f}{1-\tau}$ and hence reduces the relative savings rate s_r^f .

To decompose the reduction in relative savings rates, we use available data on average and fractile-specific income tax rates and aggregate saving rates. Saez and Zucman (2016) and Piketty, Saez and Zucman (2018) provide average tax rates for different fractiles of the wealth distribution in the US and absolute saving rates. Using their data, we construct average tax rates over the two sample periods, before and after the break. For France, we use data from Piketty and Saez (2007) who provide the tax rates and aggregate saving rates for 1970 and 2005. We use their results for the first and the second sample, respectively.

We first compute the relative tax term $\frac{1-\tau^f}{1-\tau}$, for each country, fractile and sample separately. We then calculate relative saving rates out of disposable income $s_{d,r}^f$ by using Equation (22), i.e. we use our estimated relative saving rates out of pre-tax income s_r^f and divide them by the relative tax rate term. We can furthermore calculate absolute saving rates for the respective fractile by simple multiplying the relative saving rates by the population-average saving rate in the given year. The results are shown in Table 5.

For the top 1% rather than an increase in progressivity, we find that after 1968 in

Table 5: Decomposing Estimated Relative Saving Rates

		France		US	
		1913-1967	1968-2014	1913-1982	1983-2014
Top 1%					
Relative Saving Rates					
Out of disposable income	$s_{d,r}^{T1}$	2.89	2.30	2.83	2.29
Out of pre-tax income	s_r^{T1}	2.54	2.11	2.16	1.90
Absolute Saving Rates					
Out of disposable income	s_d^{T1}	0.48	0.21	0.28	0.17
Out of pre-tax income	s^{T1}	0.40	0.18	0.19	0.12
Taxes					
Income tax rate	τ^{T1}	0.17	0.12	0.31	0.28
Relative taxes	$\frac{1-\tau^{T1}}{1-\tau}$	0.88	0.92	0.76	0.83
Top 10%					
Relative Saving Rates					
Out of disposable income	s_r^{T10}	1.82	1.65	1.84	1.60
Out of pre-tax income	s_r^{T10}	1.82	1.65	1.84	1.60
Absolute Saving Rates					
Out of disposable income	s_d^{T10}	0.31	0.15	0.20	0.12
Out of pre-tax income	s^{T10}	0.29	0.14	0.16	0.09
Taxes					
Income tax rate	τ^{T10}	0.06	0.05	0.20	0.21
Relative taxes	$\frac{1-\tau^{T10}}{1-\tau}$	0.99	0.99	0.89	0.92
Aggregate					
Saving rate (disp. inc.)	s_d	0.17	0.09	0.10	0.07
Income tax rate	τ	0.05	0.04	0.09	0.14

Note: Data for the US is from Saez and Zucman (2016) and Piketty et al. (2018b). τ available from 1916-2014, τ^{T1} and τ^{T10} from 1960-2004. Data for France is from Piketty and Saez (2007). For the first sample, we use their results for 1970, and for the second the results for 2005.

France and 1983 in the US the average tax rates at the top relative to the aggregate actually declined somewhat so that the relative tax factor increased. For the top 10% similarly the progressivity of the tax system did not increase after the structural breaks. This is in contrast to earlier increases in tax progressivity. For France, Piketty et al. (2018a) show that the rise of progressive taxation on income was a crucial determinant of the decline in wealth inequality over the 1st and 2nd World War, which is the beginning of our sample. We therefore attribute the decline of the relative savings rate to a decline of the relative savings rate out of disposable income.

Interestingly the decline in the relative savings rate at the top coincides with a decline in the aggregate savings rate in both countries. In the US the aggregate private savings rate declined from an average of 11.4% over the period 1959-1982 to 6.2% over the period 1983-2014.[20] In France, the aggregate private saving rate averaged 16.4% over the period

[20]These averages are calculated using the personal saving rate, U.S. Bureau of Economic Analysis, Personal Saving Rate [PSAVERT], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PSAVERT, July 5, 2018.

1950-1967, and averaged 15.6% in the period 1968-2014.[21]

The drop in the US that started in the early eighties was so remarkable that an earlier literature aimed to investigate the reasons behind this drop (see among others Bosworth et al. (1991), Gokhale et al. (1996), Attanasio (1998) and Parker (1999)). To identify the reasons for the declining aggregate saving rate, this literature was mainly concerned with finding the group of people which reduced saving the most. This research was however seriously hampered by the prevailing micro data at that time that could provide savings rates of different subgroups of the population only with very low precision and many ad hoc assumptions. It seems that a consensus view has never been reached. Browning and Lusardi (1996) provide an early overview of this literature and list eleven (!) different possible explanations while concluding *"[t]he variety of proposed explanations is per se an indication that there exists little consensus on what underlies the decline in saving rates."*

While this literature could not come to a conclusion of what caused the large drop, our results are in line with more recent findings of Juster et al. (2005) using PSID micro data. Juster et al. (2005) construct average saving rates for two groups of households: those that own stock in 1984 and those that do not own stock. The average savings rate for stock owners drops from 13.2% in 1984-1989 to 8.6% in 1989-1994, while those for non-stock owners barely moves from 7.7% to 7.6% (see Table 2 in Juster et al. (2005)). In other words, in Juster et al. (2005) the relative savings rate of stock owners versus non-stock owners drops from 1.71 to 1.13. Our VECM estimates suggest a drop of the relative savings rate of the top 10 % from 1.88 to 1.53. Juster et al. (2005) further show that that those households with higher capital gains in corporate equities reduced savings the most in the eighties.

The explanation of the large drop in the aggregate personal savings rate by the capital gains that households received during the stock market boom of the eighties and nineties seems plausible. As large corporate equities holdings occur mostly concentrated in households at the top of the wealth distribution, our findings of a reduced relative savings rates for the top fit nicely with their findings and provides new evidence on why the aggregate savings rate dropped so much from the early eighties onward.

5 Counterfactual Simulations of Inequality

The rise in wealth inequality in recent years begs the question of what is the relative role of rising income inequality versus movements in the relative savings rate. In this section, we aim to disentangle the historical importance of the two causal channels of wealth inequality in the past decades by performing counterfactual simulations. Let us

[21] These averages are calculated using the series Taux d'épargne des ménages, retrieved from INSEE, https://www.insee.fr/fr/statistiques/2830268, July 5, 2018. The drop in the French savings rate is realy remarkable from the eighties onwards. Average savings rate was 14.3 % over the period 1980-2014.

reconsider the wealth evolution Equation (19),

$$\Delta sh^f_{W,t+1} = -\frac{S}{W}(sh^f_{W,t} - \; s^f_r sh^f_{Y,t}) + \frac{c^f_{t+1}}{W_{t+1}} + e_t \tag{30}$$

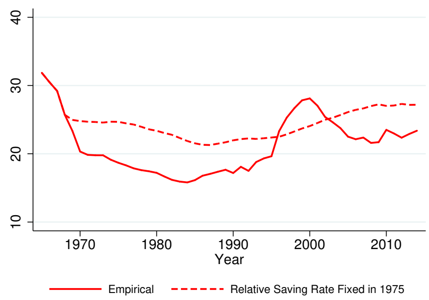
Absent large temporary deviations e_t, and churn, wealth dynamics for a certain fractile are thus mainly driven by changes in their income fraction and their relative saving rates. On the one hand, the previous VECM analysis indicates that there is a tight empirical short-run link from income to wealth concentration. Given rising income inequality in recent decades, this suggests an associated rapid increase in wealth inequality, as indeed observed empirically. On the other hand, the estimated relative saving rates seem to have declined in recent decades. The lower relative saving rates are expected to counteract the first channel of increasing income inequality, and by that attenuating the increase in wealth inequality in the long-run. In the following, we therefore aim to decompose the observed evolution in wealth inequality into fractions attributable to income inequality and relative saving rates, respectively.

Counterfactual Relative Saving Rates: We first perform counterfactual simulations of the relative saving rates at the top to gauge the extent to which the observed decrease in saving rates attenuated the first channel of increasing income inequality on the evolution of wealth inequality. We use the estimates of the VECM from the beginning of our sample in 1913 until the structural breaks in 1968 for France and 1983 for the US, respectively, as reported in Table 4. We then use the corresponding estimates of the long-run relative saving rates at the top prior to the structural break to perform out-of-sample model simulations, using the empirical income share as exogenous input. In other words, the counterfactual scenario assumes that the relative saving rates stayed constant at the 1968-level in France and the 1983-level in the US, and simulates the evolution of wealth inequality for the observed development of income inequality.

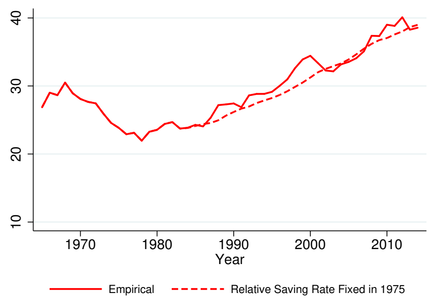
As shown in the left panels of Figure 4, the estimated decrease of relative saving rates substantially contributed to the evolution of wealth inequality in France. With relative saving rates remaining constant at their 1968-level, wealth inequality would be higher today relative to the empirical observations, at 27.1% (compared to 23.4%) and 63.4% (compared to 55.3%) for the Top 1% and Top 10%, respectively. While increases in income inequality dynamics in France were relatively subdued in recent decades, the non-negligible decline in relative saving rates also attenuated the associated upward pressure on wealth inequality. This is in contrast to the US, where the estimated relative saving rates are declined less after 1983. As a result, the counterfactual simulations produce wealth inequality dynamics largely in line with the observed increasing profile for the US, suggesting a negligible role of saving rates in recent decades. At most, the attenuating effect seems to be stronger for the Top 10% compared to the Top 1%. The results from the

Figure 4: Counterfactual Relative Saving Rates

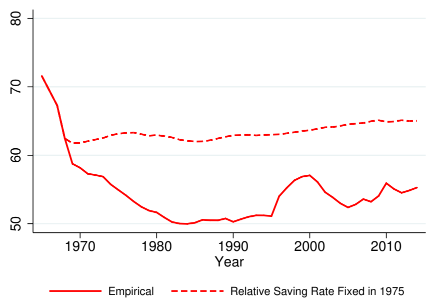
(a) France Top 1%



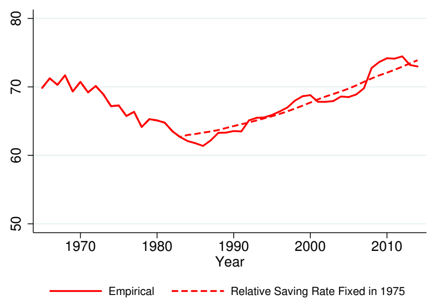
(b) US Top 1%



(c) France Top 10 %



(d) US Top 10%

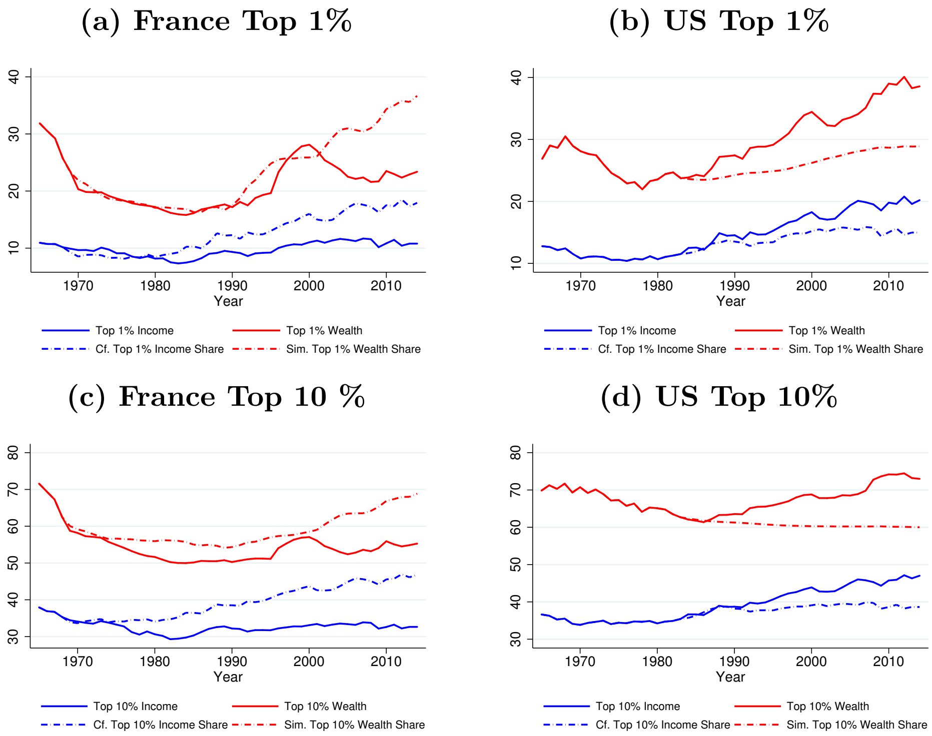


Note: Evolution of wealth inequality under constant relative saving rates after 1968 for France and 1983 for US, using VECM results from Equation (23) for the time period 1913 until the respective break.

simulation using counterfactual saving rates suggests that the observed increase in wealth inequality in the US seems to be largely attributable to the rise in income inequality. In contrast, the subdued wealth inequality rise in France appears to be a joint result of lower relative saving rates and modestly increasing income inequality.

Counterfactual Income Inequality: To provide further insights about the importance of income shares for the evolution of wealth shares, we perform a second set of counterfactual simulations. Again starting the simulations at the respective break dates, we first construct counterfactual income shares for each country using income dynamics from the other country. That is, we assume that the top income shares in France after 1968 featured the absolute changes of the corresponding top income shares in the US. We do the same vice versa for the US, starting in 1983. In other words, we obtain an increasing income inequality profile for France relative to the observed subdued dynamics, and a relatively modest rise in income inequality for US compared to actual outcomes. To give one example, this is equivalent to an increase of income concentration from 9.1% in 1975 to 18.7% in 2014 instead of 10.8% for the Top 1% in France. We use the counterfactual income inequality profile alongside the VECM estimates of post-break relative saving rates at the top to perform simulations of wealth inequality dynamics. Figure 5 shows the results.

Figure 5: Counterfactual Income Inequality



Note: Evolution of wealth inequality under counterfactual top income shares, using VECM results for the relative saving rates from Equation (23) from 1968-2014 for France and 1983-2014 for the US.

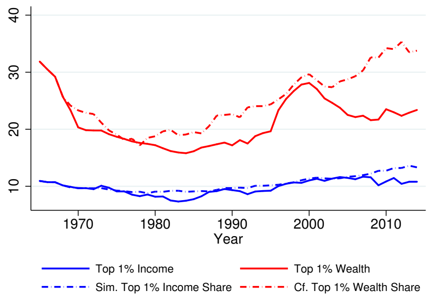
If France had experienced a rise in income inequality comparable to the US, wealth concentration would be substantial higher in recent decades. Today, it would range at 37.9% for the Top 1% and 66.3% for the Top 10%, only somewhat lower than the observed wealth inequality in the US. Notably, this simulation includes the attenuating effect of declining saving rates in France into account, such that some of the remaining difference attributable to this channel. The converse scenario applies to the US: With a roughly constant income inequality profile, wealth concentration for the Top 1% would have only risen by a bit to 25.1%, and the wealth share of the Top 10% would have decreased from 67.3% to 59.7% in 2014.

Taken together, the results from the two counterfactuals simulation reiterate one key finding of the VECM estimation as presented in the previous section: The short-run link from income inequality to wealth inequality is rapid and relatively strong. While this effect may be partially attenuated by adjustments in the relative saving rates at the top, it is sufficiently pronounced to dominate the latter channel. As such, we may think of wealth inequality as being mainly driven by income concentration, in particular in the short-run. In the long-run, the effect of relative saving rates may have a similarly important role.

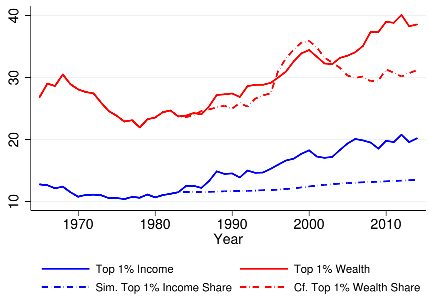
Counterfactual Wealth Inequality: The converse link from wealth inequality to income inequality is substantially weaker. We demonstrate this by performing a third set of counterfactual simulations similar in spirit to the previous one: We construct counterfactual wealth shares for both countries using wealth dynamics from the other countries. Using the estimated post-break relative saving rates at the top, we then simulate associated income inequality dynamics.

Figure 6: Counterfactual Wealth Inequality

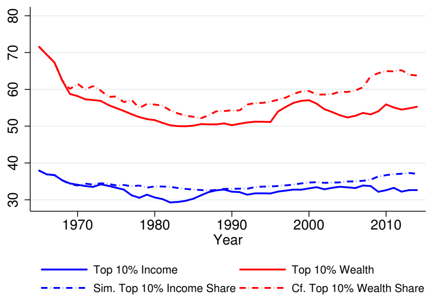
(a) France Top 1%



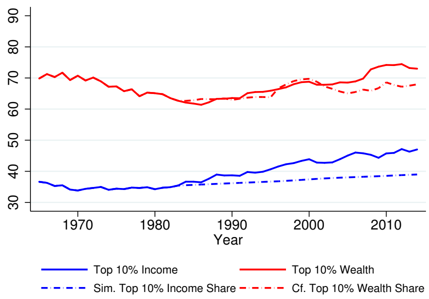
(b) US Top 1%



(c) France Top 10 %



(d) US Top 10%



Note: Evolution of income inequality under counterfactual top wealth shares, using VECM results for the relative saving rates from Equation (23) from 1968-2014 for France and 1983-2014 for the US.

As shown in Figure 6, the short- and medium-term effect of wealth concentration on income inequality is small. For France, the larger rise in wealth inequality translates into a somewhat higher income inequality towards the end of the simulation period. In the US, the lower wealth inequality translates into a relatively flat income concentration profile. Compared to the previous simulation, this again illustrates the substantial importance of income inequality in explaining wealth inequality dynamics, which the converse channel being mainly relevant over the long-run.

In terms of policy implications, we may thus conclude that policies aimed at reducing wealth inequality should tackle income inequality first. Our VECM results suggests that a given rise in income concentration leads to a rapid and substantial increase in wealth inequality. The main channel is the higher saving rate of wealthy households relative to the population average, for which we find empirical evidence. In interpreting the results, one should be reminded that we use pre-tax national income shares to measure income inequality. As such, our definition of saving rates includes the effects of taxes. As an illustration, suppose that individuals aim to save at a constant rate out of disposable income. If income taxes increase for fractile f, disposable income falls, which is mirrored by a decline in the saving rate we estimate. Against this backdrop, we may interpret the estimates of relative saving rates as including features of the tax system. In turn, we may infer that policymakers can weaken the link between income and wealth inequality by increasing the progressivity of income taxation.

However, the results from the counterfactual income inequality simulation suggest that even a substantially lower relative saving rate (for example induced by more progressive taxation) is not able to counteract a given increase in income inequality, in particular not in the short- and long-run. Therefore, one may conclude that policymakers would be well-advised to address income inequality at the pre-tax-level. Such indirect, non-tax-based policies have the potential of being associated with substantial short- and medium-run attenuation of income and wealth inequality.

6 Conclusion

We investigate the long- and short-run relationship between wealth and income inequality in the US and France over more than a century. We first show that theoretical budget equation accounting yields an error correction relationship between top wealth and top income shares. This provides the basis to estimate relative saving rates at the top as the key driver of the long-run cointegration between wealth and income inequality from the data. The empirical evidence we provide shows that top wealth and income shares are indeed cointegrated. However, we find that the relative saving rate at the top has declined after 1968 in France and after 1983 in the US, which implies a shift in the cointegration relationship. Wealth inequality in the last few decades has therefore been driven by two opposing forces. First, income inequality has increased, implying a rising wealth inequality. Second, the lowering of the relative saving rate at the top has reduced the degree to which income inequality leads to wealth inequality in the long run.

Our counterfactual analysis shows that with only modest declines in the relative saving rate in the US, most of the rise in wealth inequality can be attributed to rising income inequality. In contrast, a stronger decline in the relative saving rate in France combined with a more moderate increase in income inequality led to a much slower rise in wealth inequality. With respect to short-run dynamics, our estimates show that income inequality drives wealth inequality, while the converse link is much weaker and slower. As such, our counterfactual simulations suggest that even the substantial decline in relative saving rates observed in France would not be sufficient to considerably attenuate a rise in wealth inequality associated with a US-type rising income inequality profile.

From a broader perspective, our paper support the idea behind Piketty (2014) that a deeper understanding of the dynamics of wealth and income inequality can only come through a joint analysis of long-time series. Unfortunately, consistently defined time series measures of inequality over more than a century, such as those of WID we use here, are very rare. As such, intensifying the efforts to obtain more cross-country data, to overcome existing data gaps and to improve the overall availability and quality of inequality measures emerges as a highly worthwhile and notable contribution to inequality research, in particular for quantitative and econometric studies like the one at hand.

Our analysis also underscores the importance of the relative saving rate for the joint evolution of income and wealth inequality. In the long run, only the relative saving rates determine the gap between top wealth and income shares. Today, they are closer together than in the 1920s and 1930s. We show that the drop in the relative saving rate was not driven by changes in the progressivity of income taxation, but coincides with a drop in aggregate savings rates. Earlier research suggests that this drop might be explained by a reduced saving of those households with equity gains. Such an explanation is consistent with our finding of a drop in the relative savings rate at the top, where most equity is held.

Lastly, our results emphasize the key role of *pre-tax* income inequality in determining wealth inequality. This suggest that policymakers would be well-advised to address income inequality at the pre-tax-level. Such indirect, non-tax-based policies have the potential of being associated with substantial short- and medium-run attenuation of income and wealth inequality.

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Appendix

A Econometric Setup

Table A1: Augmented Dickey-Fuller Tests

	France				US			
Lags	Y Top 1%	W Top 1%	Y Top 10%	W Top 10%	Y Top 1%	W Top 1%	Y Top 10%	W Top 10%
0	0.36	0.23	0.33	0.56	0.59	0.36	0.80	0.69
1	0.40	0.27	0.35	0.55	0.45	0.37	0.74	0.71
2	0.46	0.29	0.39	0.58	0.50	0.32	0.69	0.57
3	0.12	0.25	0.20	0.55	0.46	0.32	0.71	0.54
4	0.18	0.22	0.22	0.54	0.68	0.51	0.75	0.42
5	0.37	0.24	0.43	0.55	0.78	0.71	0.76	0.38
6	0.20	0.28	0.25	0.56	0.68	0.57	0.77	0.28
7	0.35	0.35	0.34	0.60	0.80	0.70	0.71	0.30
8	0.37	0.37	0.43	0.62	0.80	0.66	0.53	0.38

Note: MacKinnon (1994) approximate p-values from (augmented) Dickey-Fuller tests for the top shares of pre-tax national income (Y) and net personal wealth (W). A p-value larger than 0.05 indicates that the null hypothesis that the respective variable follows a unit-root process cannot be rejected at the conventional 5%-level.

Table A2: VECM Lag Length Selection

	France		US	
IC	Top 1%	Top 10%	Top 1%	Top 10%
SBIC	1	1	1	1
HQIC	2	2	1	3
AIC	2	2	5	3

Note: Preferred lag length p for a VECM involving the respective top shares of pre-tax national income and net personal wealth according to the SBIC, HQIC and Akaike's information criterion (AIC).

B Structural Break Tests

The changed path in the historical evolution of inequality in the 1970s suggests the existence of structural breaks. When modeling the time series as following a linear time trend, univariate supremum Wald-tests strongly indicate structural breaks occurring at approximately the same time in the 1970s, as shown in Table A3. In France, the structural break occurs around 1970, while the US follows about a decade later in 1980. This provides evidence against the existence of simple linear time trends in income and wealth inequality. It also confirms that income and wealth inequality should not be seen as following trends that are merely a function of time, but strongly depend on macroeconomic circumstances and associated policies. Notably, the univariate breaks occur around the same dates as the structural breaks in relative saving rates. Table 2 from the main text is shown in the lower panel to facilitate comparison.

Table A3: Structural Breaks in Inequality

	France		US	
	Top 1%	Top 10%	Top 1%	Top 10%
Univariate Series Breaks				
Income Y	1973***	1986***	1978***	1976***
	(0.00)	(0.00)	(0.00)	(0.00)
Wealth W	1970***	1969***	1978***	1984***
	(0.00)	(0.00)	(0.00)	(0.00)
Relative Saving Rates Breaks (Table 2)				
Supremum Wald	1968***	1968***	1983***	1983***
	(0.00)	(0.00)	(0.00)	(0.00)
Cumulative sum	1968***	1968***	1983***	1983***
	(0.00)	(0.00)	(0.00)	(0.00)

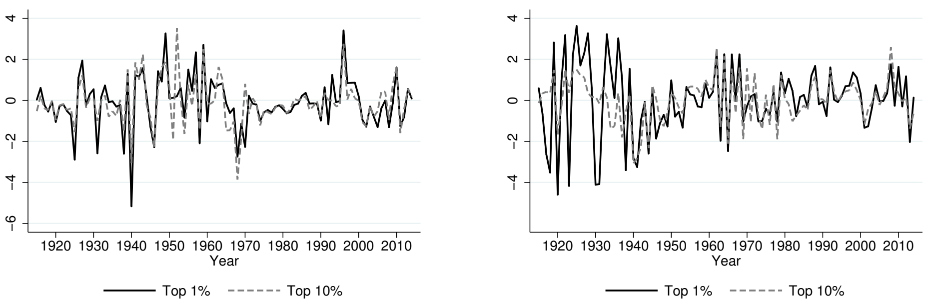
Note: Estimated break dates of relative saving rates at the top (upper panel), pre-tax national income (Y) and net personal wealth W. The univariate structural break tests are conducted using a supremum Wald test. p-values in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

C VECM Diagnostics

Post-estimation diagnosis of the VECM companion matrix eigenvalues following Johansen (1995) suggests that the cointegration relationships are stationary. However, the evidence for stability of the US Top 10% cointegration relationship is somewhat uncertain, as one eigenvalue is at 0.94, relatively close to 1. Moreover, the Jarque-Bera test strongly rejects that the residuals are normally-distributed, and for the US the Lagrange multiplier test following Johansen (1995) indicates some residual autocorrelation.

Figure A1: VECM Residuals

(a) France **(b) US**



Note: The plots show the VECM residuals resulting from the VECM given in Equation (23).

As shown in Figure A1, the VECM residuals are large at the beginning of our sample, coinciding with macroeconomic and financial market turmoil during the Great Depression and World-War II. After the 1970s, the residuals are considerably smaller, but larger again at the end of our sample during the global financial crisis. As our VECM focuses on income and wealth inequality alone, these residual properties are to be expected. They represent exogenous effects on income and wealth shares that are by design not captured in the VECM.

Table A4: Johansen Cointegration Test

	France		US	
	Top 1%	Top 10%	Top 1%	Top 10%
Trace	1	0	0	0
SBIC	0	0	0	0
HQIC	1	0	1	0

Note: Estimated VECM rank using the Johansen cointegration test with the trace statistic, Schwarz's Bayesian information criterion (SBIC) and the Hannan and Quinn information criterion (HQIC). Rank = 1 indicates a stable long-run relationship between income and wealth inequality, while rank = 0 hints at the presence of a structural break in our context.

D Sample Splits and Short-Run Adjustments

This section presents sub-sample estimates of the VECM short-run adjustment parameters. Considering the substantially smaller sample sizes, these estimates should generally be interpreted with caution. Focusing first on the short-run adjustment from income to wealth inequality, Table A5 shows the estimated coefficients for α_W. The VECM sample splits indicates that the short-run adjustment process from income to wealth inequality may have become more rapid in recent decades, in particular in France for both Top 1% and Top 10%. A similar result is detected for the Top 1% in the US, but not for the Top 10%. Overall, these results suggest that a given increase in income inequality tends to lead to faster increase in wealth inequality. For the US case, we may associate this with the decrease of the tax system progressivity induced by the Reagan tax reforms in the 1980s.

Table A5: Short-Run Adjustment from Income to Wealth Inequality α_W

	France			US		
	1913-2014	1913-1967	1968-2014	1913-2014	1913-1982	1983-2014
Top 1%	-0.09^{***}	-0.09^{**}	-0.15^{***}	-0.10^{**}	-0.06	-0.16
	(0.03)	(0.04)	(0.06)	(0.05)	(0.07)	(0.13)
Top 10%	-0.04^{**}	0.00	-0.25^{***}	-0.04^{**}	0.04	0.02
	(0.02)	(0.02)	(0.06)	(0.02)	(0.03)	(0.03)

Note: Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

The short-run effects of wealth inequality on income inequality across samples are shown in Table A6. We find some evidence that rapid short-run adjustments in this direction are mainly a pre-break feature, in particular in France, but also for the Top 10% in the US. Again keeping the caveat of the smaller sample size in mind, we may interpret the difference across samples for France as indicating the effects of the wealth tax introduced in 1989. With such a tax in effect, a given stock of wealth translates into less capital income in the next period, thus attenuating the link between wealth and income inequality.

Table A6: Short-Run Adjustment from Wealth to Income Inequality α_Y

	France			US		
	1913-2014	1913-1967	1968-2014	1913-2014	1913-1982	1983-2014
Top 1%	0.03	0.06^{**}	0.04	0.01	0.06	0.11
	(0.02)	(0.03)	(0.03)	(0.03)	(0.04)	(0.10)
Top 10%	0.03	0.05^{**}	0.01	0.01	0.09^{***}	0.10^{***}
	(0.02)	(0.02)	(0.04)	(0.02)	(0.03)	(0.03)

Note: Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.